Problem Set I

We start from

$$V_s = \left(\frac{(\gamma+1)KE_0}{4\pi\rho_1 R_s^3}\right)^{1/2} . \tag{1}$$

We can rewite this equation as:

$$R_s^{3/2} dR_s = \left(\frac{(\gamma+1)KE_0}{4\pi\rho_1}\right)^{1/2} dt \tag{2}$$

Which we integrate to get

$$R_s == \left(\frac{25KE_0}{6\pi\rho_1}\right)^{1/5} t^{2/5} \tag{3}$$

Where we used the fact that γ was 5/3. R_s here is in cm, E_0 in ergs, ρ_1 in g cm^{-3} and t in seconds.
Now:

E_0 ergs $= E_{51}$ x 10^{51} ergs
$t = t_4$ x 10^4 x (3600x24x365) seconds
$\rho_1 = n_1$ x $m_{\mathrm{H}} = 1.67410^{-24}\ n_1$

So

$$R_s(\mathrm{cm}) = \left(\frac{2510^{51}10^8(3600x24x365)^2}{6\pi 1.67410^{-24}}\right)^{1/5} \left(\frac{KE_{51}}{n_1}\right)^{1/5} t_4^{2/5} \tag{4}$$

Which translates to

$$R_s(\mathrm{cm}) = 3.795810^{19} \left(\frac{KE_{51}}{n_1}\right)^{1/5} t_4^{2/5} \tag{5}$$

which gives:

$$R_s(\mathrm{pc}) = 12.4 \left(\frac{KE_{51}}{n_1}\right)^{1/5} t_4^{2/5} \tag{6}$$

Problem set II

N=nV (number of source is the number density times the volume).
Assuming homogeneous distribution, n is constant as a function of the distance and N is then proportional to d^3 (because V=$4/3\pi d^3$).

Now (Luminosity) and (Flux) are related by the relation $L = F \times 4\pi d^2$ which means that d is proportional to the (luminosity/flux) to the power 1/2.
So at a given flux, the number of sources which appear brighter that a luminosity L is proportional to $L^{3/2}$.